ECE 313: Electromagnetic Waves

Lecture 7: fields in Dielectric and Good conductors

Lecturer :Dr. Gehan Sami

Wave propagation in dielectric:

$$\nabla^2 \overline{E} + \gamma^2 \overline{E} = 0$$

$$\gamma^2 = -j\omega\mu\sigma + \omega^2 \mu\varepsilon \quad \text{or}$$

$$j\gamma = j\sqrt{\omega^2 \mu\varepsilon - j\omega\mu\sigma} = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{j\sigma}{\omega\varepsilon}} = \alpha + j\beta$$

$$\gamma = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{j\sigma}{\omega\varepsilon}} \quad \boxed{loss\ due\ condictivity\ \sigma}\ (with\ real\ \varepsilon)$$

$$losses\ \exp ressed\ through\ complex\ permitivity\ \varepsilon_c = \varepsilon' - j\varepsilon''$$

$$\gamma = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\varepsilon''}{\varepsilon'}}$$

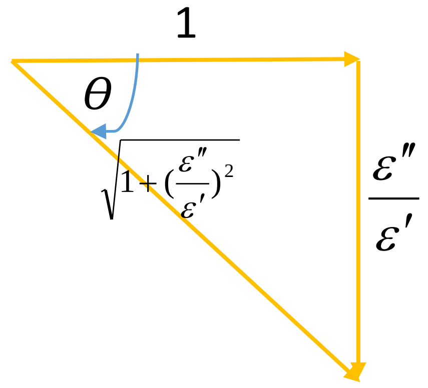
$$comparing: \varepsilon'' = \frac{\sigma}{\omega}\ and\ \varepsilon' = \varepsilon_0\varepsilon_r$$

Ratio $\varepsilon''/\varepsilon'$ is called loss tangent
$\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon = \tan\delta$
δ called loss angle

wave propagation in dielectric:

$$j\gamma = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \alpha + j\beta$$

$$j\gamma = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\varepsilon''}{\varepsilon'}} = j\omega\sqrt{\mu\varepsilon'}\sqrt{\sqrt{1+(\frac{\varepsilon''}{\varepsilon'})^2}}\,(\cos(\theta/2) - j\sin(\theta/2))$$



$$\cos(\theta/2) = \sqrt{0.5}\sqrt{\frac{1}{\sqrt{(1+(\frac{\varepsilon''}{\varepsilon'})^2)}} + 1} \qquad \sin(\theta/2) = \sqrt{0.5}\sqrt{1 - \frac{1}{\sqrt{(1+(\frac{\varepsilon''}{\varepsilon'})^2)}}}$$

$$\cos\theta = 2\cos(\theta/2)^2 - 1 = 1 - 2\sin(\theta/2)^2$$

$$j\gamma = j[\,\omega\sqrt{\frac{\mu\varepsilon'}{2}}\sqrt{1+\sqrt{1+(\frac{\varepsilon''}{\varepsilon'})^2}} - j\omega\sqrt{\frac{\mu\varepsilon'}{2}}\sqrt{\sqrt{1+(\frac{\varepsilon''}{\varepsilon'})^2} - 1}\,]$$

$$j\gamma = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\sqrt{\sqrt{1+(\frac{\varepsilon''}{\varepsilon'})^2} - 1} + j\omega\sqrt{\frac{\mu\varepsilon'}{2}}\sqrt{1+\sqrt{1+(\frac{\varepsilon''}{\varepsilon'})^2}} = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}}\sqrt{\sqrt{1+(\frac{\sigma}{\omega\varepsilon_0\varepsilon_r})^2} - 1}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}}\sqrt{\sqrt{1+(\frac{\sigma}{\omega\varepsilon_0\varepsilon_r})^2} + 1}$$

Example:

A sinusoidal electric intensity of amplitude 250 (V/m) and frequency 1 (GHz) exists in a lossy dielectric medium that has a relative permittivity of 2.5 and a loss tangent of 0.001. Find the average power dissipated in the medium per cubic meter.

Solution First we must find the effective conductivity of the lossy medium:

$$\tan \delta_c = 0.001 = \frac{\sigma}{\omega \epsilon_0 \epsilon_r},$$

$$\sigma = 0.001(2\pi 10^9)\left(\frac{10^{-9}}{36\pi}\right)(2.5)$$

$$= 1.39 \times 10^{-4} \text{ (S/m)}.$$

$\varepsilon_0 = 8.85 * 10^{-12} = 10^{-9}/(36\pi)$

The average power dissipated per unit volume is

$$p = \tfrac{1}{2}JE = \tfrac{1}{2}\sigma E^2$$

$$= \tfrac{1}{2} \times (1.39 \times 10^{-4}) \times 250^2 = 4.34 \quad \text{(W/m}^3\text{)}.$$

Example 11.4

consider plane wave propagation in water, at microwave frequency of 2.5 GHz.
At frequencies in this range and higher. Real and imaginary parts of the permittivity are present., and both vary with frequency. ϵ'' that increases with increasing frequency, ϵ' decreases with increasing frequency. At 2.5 GHz, $\epsilon'_R = 78$ and $\epsilon''_R = 7$

$$\alpha = \frac{(2\pi \times 2.5 \times 10^9)\sqrt{78}}{(3.0 \times 10^8)\sqrt{2}} \left(\sqrt{1 + \left(\frac{7}{78}\right)^2} - 1 \right)^{1/2} = 21 \text{ Np/m}$$

similar to that for α, we find $\beta = 464$ rad/m. The wavelength is $\lambda = 2\pi/\beta = 1.4$ cm, whereas in free space this would have been $\lambda_0 = c/f = 12$ cm.

$$\eta = \frac{377}{\sqrt{78}} \frac{1}{\sqrt{1 - j(7/78)}} = 43 + j1.9 = 43\angle 2.6^\circ \ \Omega$$ and E_x leads H_y in time by 2.6° at every point.

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon(1 - j\frac{\varepsilon''}{\varepsilon})}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \sqrt{\frac{1}{1 - j\frac{\varepsilon''}{\varepsilon}}} \qquad \text{where} \qquad \varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon', \frac{\varepsilon''}{\varepsilon} = \frac{\varepsilon''_r}{\varepsilon_r}$$